

Important Notice:

- ♣ The answer paper must be submitted before the deadline.
- ♠ The answer paper MUST BE sent to the CU Blackboard. Please refer to the course web for details.
- ✂ Each answer paper must include your name and student ID.

1. Let X be a Hilbert space and let $T : X \rightarrow X$ be a bounded operator. Show that $\lim_{\lambda \rightarrow \infty} \|(T - \lambda)^{-1}\| = 0$, where $\lambda > \|T\|$.
2. Using the notation given as in Question (1), let $f(t) := t^n + a_{n-1}t^{n-1} + \cdots + a_1t + a_0$ be a complex polynomial. Put $f(T) := T^n + a_{n-1}T^{n-1} + \cdots + a_1T + a_0I \in \mathcal{L}(X)$.
 - (i) Show that $f(T)$ is invertible in $\mathcal{L}(X)$ if and only if $\alpha \notin \sigma(T)$ for all roots α of f . (Hint: use the Fundamental Theorem of Algebra).
 - (ii) Let f be a polynomial as in Part (i). Show that if $\lambda_0 \in \sigma(T)$, then $f(\lambda_0) \in \sigma(f(T))$.

*** End ***